



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/33

Paper 3 Further Mechanics

May/June 2026

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen. Do **not** use correction fluid or tape.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use 10 m s^{-2} .

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.



- 1 A particle P of mass m is moving in a horizontal circle with constant angular speed ω on the smooth inner surface of a hemispherical shell of radius a . The plane of the circular motion is at a height $h = \frac{1}{3}a$ above the lowest point on the shell. The angle between the vertical and the normal reaction of the surface on P is θ .

Find ω in terms of a and g .

[5]

[illegible]



- 2 A particle P of mass m is moving in a straight horizontal line towards a fixed point A on the line. The particle is subject to a resistive force that is inversely proportional to the square of the distance between P and A . Initially, the distance between P and A is d and the speed of P is u . When the distance between P and A is $\frac{1}{2}d$, the particle has lost $\frac{3}{4}$ of its kinetic energy.

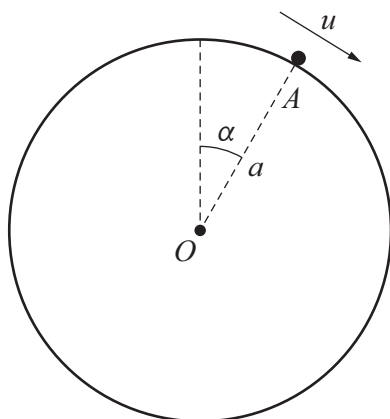
Find, in terms of d , the minimum distance of P from A .

[6]

[illegible]



3

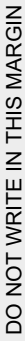


A fixed smooth solid sphere has centre O and radius a . A particle of mass m is projected perpendicular to OA in a downward direction with speed u from the point A on the outer surface of the sphere, where OA makes an angle α with the upward vertical through O (see diagram).

The particle moves in part of a vertical circle on the surface of the sphere. It loses contact with the sphere at the point B , where OB makes an angle β with the upward vertical through O . The speed of the particle at the point B is v .

Given that the ratio of u to v is $2 : 3$ find the ratio $\cos \alpha : \cos \beta$. [5]

[illegible]



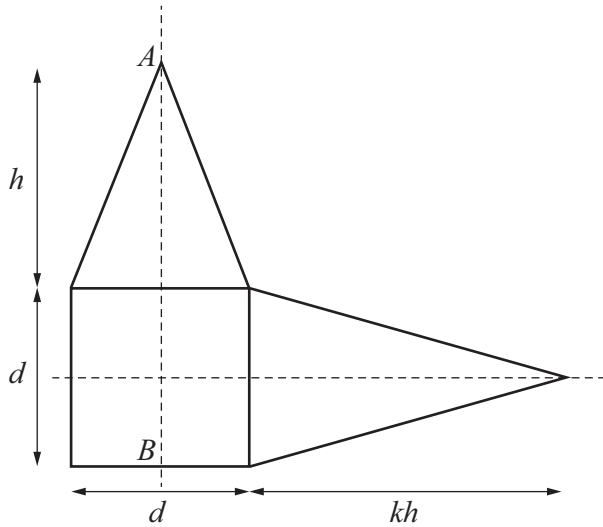
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An object consists of a uniform solid cube of side length d , and two uniform solid square-based pyramids. The cube and the pyramids are made of the same material.

One pyramid, of base length d and height h , is fixed to the top face of the cube so that the base of this pyramid and the face of the cube coincide. Point A is at the top vertex of this pyramid. Point B is in the centre of the bottom face of the cube, vertically below A . The second pyramid, of base length d and height kh , is fixed to a vertical face of the cube so that the base of this pyramid and the vertical face of the cube coincide (see diagram, which shows a cross-section, including the axes of symmetry).

- (a) Find an expression for the distance of the centre of mass of the object from AB in terms of h , k , and d .

[4]

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The object is placed on a horizontal plane so that the bottom face of the cube that contains point B is in contact with the plane. It is given that the object is on the point of toppling.

- (b) Find the value of h in terms of k and d . [3]

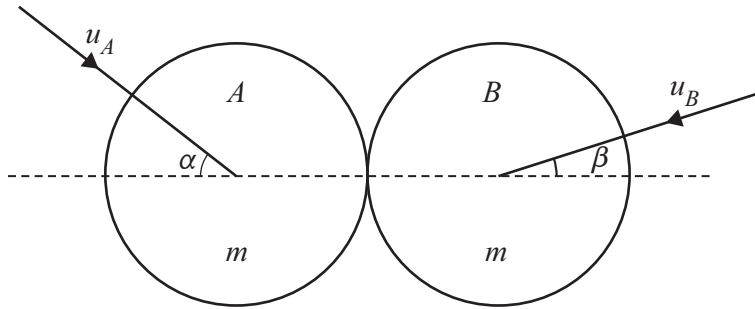
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- (c) Given that $h = 1.5d$, find the value of k . [2]

[illegible]



5



Two uniform smooth spheres A and B of equal radii and equal masses m are moving with speeds u_A and u_B respectively on a smooth horizontal surface when they collide. Immediately before the collision, the direction of motion of A makes an angle α with the line of centres, and the direction of motion of B makes an angle β with the line of centres (see diagram). Immediately after the collision, the direction of motion of A makes an angle θ with the line of centres, and the direction of motion of B makes an angle φ with the line of centres. It is given that $\cos \alpha = \cos \varphi = \frac{4}{5}$ and $\cos \beta = \frac{24}{25}$ and $\cos \theta = \frac{3}{5}$.

- (a) Find the percentage loss in the kinetic energy of A .

[2]

[illegible]



(b) Find the value of the coefficient of restitution between the spheres.

[4]

[illegible]



- 6 A smooth plane is inclined at an angle θ to the horizontal, where $\sin \theta = \frac{4}{5}$. One end of a light elastic spring, with natural length $10a$ and modulus of elasticity $1.6mg$, is attached to a fixed point O on the plane. The other end of the spring is attached to a particle P of mass $0.2m$. The point O and the particle P lie on the same line of greatest slope of the plane, with P above O . The particle P is at rest.

(a) Show that the distance OP is $9a$. [3]

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A particle Q of mass $0.3m$ is held on the plane on the line of greatest slope through O , at a distance $10a$ above P . The particle Q is released from rest. The particle Q then collides and coalesces with particle P .

(b) Find, in terms of a and g , the speed of the combined particle immediately after the collision. [4]

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- (c) As a result of the collision, the spring is further compressed. Find, in terms of a , the minimum distance of the combined particle from O . [4]

[illegible]



- 7 At time $t = 0$ s, a particle P is projected from a point O on horizontal ground. The initial velocity of P is 25 m s^{-1} at an angle α above the horizontal, where $\sin \alpha = \frac{4}{5}$. At time $t = T$ s, a second particle Q is projected vertically upwards from the horizontal ground directly below the point where P will reach its greatest height. The initial velocity of Q is also 25 m s^{-1} . Both particles move freely under gravity. The two particles collide when Q is travelling upwards and at the point where P reaches its greatest height.

(a) Show that the two particles collide 2 seconds after P is projected. [1]

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(b) Find the value of T . [3]

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- (c) As a result of the collision, the trajectory of P changes, but the horizontal component of its velocity remains unchanged. Immediately after the collision, the speed of P is $v \text{ ms}^{-1}$, at an angle θ above the horizontal.

Find, in terms of $\tan \theta$, the greatest height of P above the ground after the collision. [4]

[illegible]

[illegible]





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