



Cambridge International AS & A Level

 CANDIDATE
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FURTHER MATHEMATICS

9231/31

Paper 3 Further Mechanics

May/June 2026
1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen. Do **not** use correction fluid or tape.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use 10 m s^{-2} .

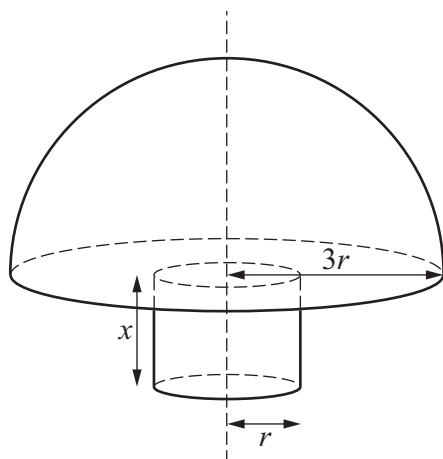
INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

 This document has **16** pages.



1



An object consists of a uniform solid hemisphere of radius $3r$ attached to a uniform solid cylinder of radius r and height x . The hemisphere and the cylinder are made of the same material. The axes of symmetry of the hemisphere and the cylinder coincide (see diagram).

- (a) Find, in terms of r and x , the distance of the centre of mass of the object from the base of the cylinder.

[4]

[illegible]



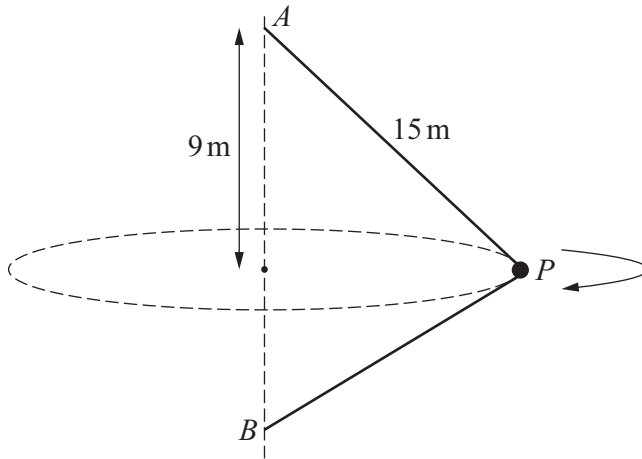
- (b) The point A lies on the circumference of the base of the hemisphere. When the object is freely suspended from point A , the base of the hemisphere makes an angle θ with the downward vertical through A .

It is given that $x = r$.

Find the value of θ .

[3]

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The point B is vertically below the point A . One end of a light inextensible string is attached to A , and the other end is attached to B . A particle P of mass 7 kg is attached to the string so that the length AP is 15 m . The particle P moves at a constant angular speed of 4 radians per second in a horizontal circle with centre between A and B at a distance 9 m vertically below A (see diagram). The string has length 28 m , and both sections of the string are taut.

The tensions in the sections of the string AP and BP are T_A and T_B respectively.

Find the value of T_A and the value of T_B .

[5]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



- 3** A particle P of mass 6.4 kg moves along a straight horizontal line. At time $t\text{ s}$, the speed of P is $v\text{ m s}^{-1}$ and the distance of P from its starting position is $x\text{ m}$. The forces acting on P are a constant driving force of magnitude 8 N and a resistive force of magnitude $0.5v^2\text{ N}$. Initially, P is at rest.

(a) Find the speed of P when it has travelled 0.8 m.

[5]

[illegible]



(b) Find an expression for t in terms of v .

[4]

[illegible]



- 4 A particle P is moving on a fixed smooth horizontal surface with speed u . The particle collides with a fixed smooth vertical barrier. Immediately before the collision, the direction of motion of P makes an angle α with the barrier, where $\tan \alpha = \frac{12}{5}$.

The kinetic energy of P after the collision is $\frac{4}{13}$ times its kinetic energy before the collision.

- (a) Find the coefficient of restitution between P and the barrier. [4]

[illegible]



- (b) A second smooth vertical barrier is fixed to the surface and to the first barrier. The angle between the two barriers is 120° . Particle P subsequently collides with the second barrier and rebounds immediately after with speed w at an angle θ with the second barrier. The coefficient of restitution is the same as in the first collision.

Find the value of θ correct to 3 significant figures.

[4]

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- 5** A particle of mass m is attached to a fixed point O by a light inextensible string of length a . The particle initially hangs directly below O at the point A . The particle is then projected horizontally with speed $2\sqrt{ag}$, and begins to move in a vertical circle.
- (a)** Find an expression for the tension in the string when it has turned through an angle θ . Give your answer in terms of m , g and θ . [4]

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- (b)** Find, in terms of a , the maximum height above the horizontal through A that is reached by the particle.

[5]

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- 6** A light elastic string has natural length 2.5 m and modulus of elasticity 15 N. One end of the string is attached to the fixed point O , and the other end is attached to a particle P of mass 1.5 kg. The particle P is held at O and then released from rest so that it begins to fall vertically. In its subsequent motion, P moves with velocity $v \text{ m s}^{-1}$ when the extension of the string is $x \text{ m}$. The maximum extension of the string during this motion is $E \text{ m}$.

(a) Show that, for $0 < x < E$,

$$v^2 = -4(x^2 - 5x - 12.5). \quad [3]$$

[illegible]



(b) Find the value of E .

[1]

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(c) Find the maximum speed of P .

[2]

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- 7 The points O and A lie on a horizontal plane 10 m apart. The point A lies at the foot of another plane which is inclined at an angle of α above the horizontal such that $\tan \alpha = \frac{4}{3}$. The point B is on the inclined plane at a distance of 5 m from A measured along a line of greatest slope. The points O , A and B are in the same vertical plane and angle OAB is obtuse.

A particle is projected from O at an angle θ to the horizontal with initial speed $u \text{ ms}^{-1}$ towards the inclined plane. The particle moves freely under gravity. The particle strikes the inclined plane at the point B when its direction of motion is perpendicular to the inclined plane.

Find, in either order, the value of u and the value of θ .

[6]

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[illegible]



Additional page

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