



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/14

Paper 1 Further Pure Mathematics 1

May/June 2026
2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen. Do **not** use correction fluid or tape.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

 This document has **16** pages. Any blank pages are indicated.



- 1 (a)** Use standard results from the list of formulae (MF19) to show that

$$\sum_{r=1}^n (r-1)^2 = an^3 + bn^2 + cn,$$

where a , b and c are constants to be determined.

[2]

[illegible]



It is given that $u_r = \frac{1}{r^2 - 2r + 1} - \frac{1}{r^2 + 2r + 1}$.

(b) Use the method of differences to find $\sum_{r=2}^n u_r$ in terms of n . [4]

[illegible]

(c) State the value of $\sum_{r=2}^{\infty} u_r$. [1]

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2 The equation $2x^4 + 3x^3 + x + 1 = 0$ has roots $\alpha, \beta, \gamma, \delta$.

(a) Find a quartic equation whose roots are $\frac{1}{\alpha^3}, \frac{1}{\beta^3}, \frac{1}{\gamma^3}, \frac{1}{\delta^3}$ and state the value of $\frac{1}{\alpha^3} + \frac{1}{\beta^3} + \frac{1}{\gamma^3} + \frac{1}{\delta^3}$.

[5]

(b) Find the value of $\frac{1}{\alpha^6} + \frac{1}{\beta^6} + \frac{1}{\gamma^6} + \frac{1}{\delta^6}$.

[2]





3 The sequence of positive numbers $u_1, u_2, u_3, \dots$ is such that $u_1 > 1$ and, for $n \geq 1$,

$$u_{n+1} = \frac{u_n^2 + 2u_n + 2}{u_n + 4}.$$

(a) By considering $u_{n+1} - 1$, or otherwise, prove by mathematical induction that $u_n > 1$ for all positive integers n .

[5]

(b) Show that $u_{n+1} < u_n$ for $n \geq 1$.

[3]



4 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 1 & k & -1 \\ 3 & 1 & k \\ 3 & 1 & -1 \end{pmatrix},$$

where k is a real constant.

(a) Find the values of k for which $\mathbf{A}$ is singular. [3]

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The matrices $\mathbf{B}$ and $\mathbf{C}$ are given by

$$\mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \\ 1 & 0 \end{pmatrix} \text{ and } \mathbf{C} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}.$$

(b) Show that $\mathbf{CAB} = \begin{pmatrix} k+3 & -1 \\ 2 & -k-1 \end{pmatrix}$. [3]

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(c) It is given that $k = -2$

Show that the transformation in the x - y plane represented by $\mathbf{CAB}$ has no invariant lines. [5]

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5 The points A, B, C have position vectors

$$4\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}, \quad 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}, \quad 5\mathbf{i} + 2\mathbf{j} + 2\mathbf{k},$$

respectively, relative to the origin O .

(a) Find the equation of the plane ABC , giving your answer in the form $ax + by + cz = d$. [5]

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The point D has position vector $2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$.

(b) Find the acute angle between the line OD and the plane ABC . [3]

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(c) Find the shortest distance between the lines AB and CD .

[5]

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- 6 The curve C has equation $y = f(x)$, where $f(x) = \frac{2x^2 - x + 1}{2x^2 - 2x}$.

(a) Find the equations of the asymptotes of C .

[2]

[illegible]

- (b)** Find the coordinates of any stationary points on C , giving your answers correct to 1 decimal place.

[4]

[illegible]



(c) Sketch C .

[3]

(d) Sketch the curve with equation $y = \frac{1}{f(x)}$ and state the set of values for which $\frac{1}{f(x)} < f(x)$. [4]

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- 7 (a) Show that the curve with Cartesian equation

$$(x^2 + y^2)^{\frac{3}{2}} = xy$$

has polar equation $r = \frac{1}{2} \sin 2\theta$. [3]

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The curves C_1 and C_2 have polar equations

$$r = \frac{1}{2} \sin 2\theta \quad \text{and} \quad r^2 = \frac{1}{\left(\theta - \frac{1}{4}\pi\right)^2 + 5}$$

respectively, where $0 \leq \theta \leq \frac{1}{2}\pi$.

- (b) Show that C_1 and C_2 intersect where

$$\left(\left(\theta - \frac{1}{4}\pi\right)^2 + 5\right) \sin^2 2\theta = 4$$

and verify by calculation that this equation has a root α between $\theta = 0.5$ and $\theta = 0.6$. [3]

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- (c) Sketch C_1 and C_2 on the same diagram, clearly identifying each curve and any lines of symmetry.

[4]

- (d) The region R is enclosed by C_1 and C_2 and includes the pole. Show that the area of R is given by

$$\frac{1}{32}(4\alpha - \sin 4\alpha) + \frac{1}{5}\sqrt{5} \tan^{-1}\left(\frac{1}{5}\sqrt{5}\left(\frac{1}{4}\pi - \alpha\right)\right),$$

where α is the root found in 7(b).

[6]

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Additional page

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