

Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/44**

Paper 4 Further Probability & Statistics

May/June 2026**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme contains the instructions and marking guidance that our examiners used to mark candidate responses for this component.

Before they start marking, examiners complete a standardisation process. They review a range of candidate responses and discuss the mark scheme in detail to agree which answers can and cannot be accepted. Examiners award marks where it is clear that candidate responses have met the requirements of the mark scheme. These requirements may vary between different components or different versions of the same component, depending on the exact requirements of the question and the candidate responses seen during the standardisation process.

Sometimes, our mark schemes may allow marks to be awarded to responses which contain elements that are factually incorrect or for content not covered by the syllabus. We do this in response to the demands of a specific question to support candidates and make sure that our assessments are fair. However, these allowances should not be used as a guide for teaching and learning.

We cannot discuss the mark scheme with schools, and we recommend that schools read the mark scheme together with the assessment material and the Principal Examiner Report for Teachers.

This document consists of **18** printed pages.

General marking principles

All examiners must apply these marking principles when marking candidate responses.

1	Examiners must award marks for each response in line with: - the specific content defined in the mark scheme - the specific skills defined in the mark scheme or in the level descriptions - the standard of response required as exemplified by the standardisation scripts.
2	Examiners must award whole marks (not half marks, or other fractions).
3	Examiners must award marks positively . This means that: - marks are not deducted for errors or omissions - marks are awarded for correct/valid answers as defined in the mark scheme and also for other valid answers which go beyond the scope of the syllabus and mark scheme referring to your team leader as appropriate - the quality of spelling, punctuation and grammar are judged only when the mark scheme indicates that these features are specifically assessed in the question. However, the meaning of all responses must be unambiguous.
4	Examiners must consistently apply the requirements of the mark scheme and any rules for what to do when candidates have not followed instructions correctly.
5	Examiners must use the full range of marks defined in the mark scheme, unless limited by the quality of the candidate responses seen.
6	Examiners must award marks according to the mark scheme and must not award marks with grade thresholds or grade descriptions in mind.

Mathematics Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (ISW).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number or sign does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case

Annotation	Meaning
	Indicates that work/page has been seen
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

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The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	$s_x^2 = \frac{934.3}{39} [= 23.956]$ $s_y^2 = \frac{756.2}{59} [= 12.817]$	B1	Either expression, may be embedded. Accuracy must be 3 significant figures or better. May be implied by 17.25, $\frac{69}{4}$, 0.71875, $\frac{23}{32}$, $\frac{1}{8}\sqrt{46}$ or 0.84779 seen. Allow B1 for correct form of pooled estimate.
	$s^2 = \frac{'23.956'}{40} + \frac{'12.817'}{60} [= 0.8125]$	M1	Correct form using <i>their</i> values. M0 for pooled estimate.
		A1	For fully correct expression SOI. Implied by AWRT 0.812 or 0.813 or $s = 0.901$ or fully correct interval.
	$\frac{67.1}{40} - \frac{84.1}{60} \pm 1.96\sqrt{0.8125} [= 0.2758 \pm 1.7667]$	M1	Correct form with any z-value. Condone $t = 1.98$. FT <i>their</i> s . If pooled must use $\sqrt{s_p^2 \left(\frac{1}{40} + \frac{1}{60} \right)}$.
		B1	1.96 seen.
	$[-1.49, 2.04]$	A1	CWO Accept with inequality signs or open brackets. Condone $[2.04, -1.49]$. Do not accept 0.276 ± 1.767 .
		6	Use of pooled s scores max 3/6 B1 M0 A0 M1 B1 A0 leading to $[-1.39, 1.94]$.

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Question	Answer	Marks	Guidance
2(a)	$\left[G'_X(t) = \frac{1}{6}(t^2 + 2) + \frac{2}{6}t(t+1) = \frac{1}{6}(3t^2 + 2t + 2) \right]$ $[E(X) = \frac{7}{6}]$	B1	AWRT 1.17
	$[G''_X(t) = t + \frac{1}{3}]$ $G''_X(1) = \frac{4}{3}$	M1	Attempt to differentiate their $G'_X(t)$ and evaluate at $t = 1$. May be implied by an expression for $G''_X(1)$ or $\frac{4}{3}$ seen.
	$\text{Var}(X) = \frac{4}{3} + \frac{7}{6} - \left(\frac{7}{6}\right)^2$	M1	Use correct formula with <i>their</i> values.
	$\frac{41}{36} [= 1.14]$	A1	AWRT 1.14, NFWW (for example, an incorrect $G''_X(t)$, with $G''_X(1) = \frac{4}{3}$ scores max B1 M1 M1 A0 XP).
		4	

Question	Answer	Marks	Guidance
2(b)	EITHER: $G_Y(t) = \left(\frac{1}{6}(t+1)(t^2+2) \right)^3 \left[= \left(\frac{1}{6}(t^3+t^2+2t+2) \right)^3 \right]$	(*M1)	SOI, must be cubed.
	$\frac{1}{216}(8+24t+36t^2+\dots) \left[= \frac{1}{27} + \frac{1}{9}t + \frac{1}{6}t^2 + \dots \right]$	DM1	Attempt to expand to form a polynomial up to at least t^2 . May be unsimplified. May be implied by a sum of coefficients or AWRT 0.315. ISW with powers of t that are 3 or above.
	$P(Y \leq 2) = \frac{8}{216} + \frac{24}{216} + \frac{36}{216} = \frac{1}{27} + \frac{1}{9} + \frac{1}{6} = \frac{17}{54} \left[= 0.315 \right]$	A1)	AWRT 0.315
	OR: Alternative method for X_1, X_2, X_3 : $P(\text{all zero}) = \left(\frac{1}{3} \right)^3, P(\text{two 0s, one 1}) = 3 \times \left(\frac{1}{3} \right)^2 \times \left(\frac{1}{3} \right)$ $P(\text{two 0s, one 2}) = 3 \times \left(\frac{1}{3} \right)^2 \times \left(\frac{1}{6} \right), P(\text{one 0, two 1s}) = 3 \times \frac{1}{3} \times \left(\frac{1}{3} \right)^2$	(M1)	Consider all 4 cases.
		M1	Correct probability calculations for at least 2 cases. Only 2 cases considered can score a maximum of M0 M1 A0.
	$\frac{17}{54}$	A1)	AWRT 0.315, CWO
	Available marks	3	

Question	Answer	Marks	Guidance
3(a)	$\left[X \sim N(236.5, 11.6^2) \right]$ $\left[c = 120 \times P(250 \leq X \leq 260) = \right] 12.10$	B1	12.09 or 12.10 or 12.11 Accept 12.1 for 12.10.
	$\left[d = 120 \times (1 - \Phi(2.026)) = \right] 2.57 \text{ or } [(120 - \text{total}) =] 2.56$	B1	2.55, 2.56 or 2.57.
		2	

Question	Answer	Marks	Guidance
3(b)	H_0: A normal distribution fits the data. H_1: A normal distribution does not fit the data. OR H_0: A normal distribution is a satisfactory/suitable model for the masses (of this species of frog). H_1: A normal distribution is not a satisfactory/suitable model for the masses (of this species of frog).	B1	Condone $N(236.5, 11.6^2)$ included in hypotheses. Must mention distribution/model and data/masses.
	Combines expected frequencies in either first two cells or last two cells.	M1	At least one cell must have an expected frequency less than 5 seen. May be implied by a corresponding contribution seen.
	$\frac{(11 - 9.28)^2}{9.28} + \frac{(20 - 25.22)^2}{25.22} + \dots + \frac{(10 - '14.67')^2}{'14.67'}$ $[0.3188 + 1.0804 + 0.0134 + 2.5404 + 1.4866]$ OR $\frac{11^2}{9.28} + \frac{20^2}{25.22} + \dots + \frac{10^2}{'14.67'} - 120$ No combining gives $[0.3251 + 0.1415 + 1.0804 + 0.0134 + 2.5404 + 1.3893 + 0.1225]$	M1	At least 2 expressions (not including 120) of correct form seen. May see individual terms not summed. May be implied by 2 contributions correct to 3 decimal places seen. May be implied by AWR 5.43 or 5.44 or 5.45. For contribution from combining last two cells allow AWR 1.48 or 1.49 to 3 significant figures. For last two cells: 12.09 gives 1.3836, 2.57 gives 0.1264.
	5.440	A1	AWR 5.43 or 5.44 or 5.45
	Critical value is 5.991	B1	With both first 2 cells and last 2 cells combined.
	'5.440' < 5.991, accept H_0 / do not reject H_0 / not significant.	M1	Compare <i>their</i> 5.44 with 5.991 or 7.815 or 9.488 or 11.07 or 12.59 and appropriate conclusion (may be in terms of H_1). Test result can be implied by an attempt at an appropriate conclusion in context ignoring hypotheses.

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Question	Answer	Marks	Guidance
3(b)	Insufficient evidence to suggest the masses (of this species of frog) are not normally distributed. OR Insufficient evidence to reject the biologist's belief. OR Insufficient evidence to suggest that a normal distribution is not a suitable model / good fit (for the data).	A1	Correct conclusion from correct working (using 5.99) ignoring hypotheses. In context. Level of uncertainty in language used (for example, not 'prove'). Allow 'Not enough evidence / no sufficient evidence that / to show / to conclude that ...'. Do not accept 'No evidence...'. Do not accept statements such as "there is sufficient evidence to suggest...".
		7	

Question	Answer	Marks	Guidance
4(a)	Reduces the uncertainty/variability from choosing cyclists of different abilities. OR Controls for differences between different cyclists.	B1	Allow for an attempt at an explanation that pairing/differences/matching leads to improved accuracy/precision/clarity/sensitivity/certainty.
		1	

Question	Answer	Marks	Guidance										
4(b)	H ₀ : population median difference = 0 H ₁ : population median difference > 0	B1	Allow $m, m_d, m_a - m_c$ OE but not μ . If m defined, must see population . Median not mean or average.										
	<table><tr><td>3.9</td><td>3</td><td>8</td><td>−1.4</td><td>1.8</td><td>3.7</td><td>0.1</td><td>0.2</td><td>−0.3</td><td>2.9</td></tr></table>	3.9	3	8	−1.4	1.8	3.7	0.1	0.2	−0.3	2.9	M1	Attempt at (signed) differences. At most 2 errors. May be seen with table before 4(a) .
	3.9	3	8	−1.4	1.8	3.7	0.1	0.2	−0.3	2.9			
	<table><tr><td>9</td><td>7</td><td>10</td><td>−4</td><td>5</td><td>8</td><td>1</td><td>2</td><td>−3</td><td>6</td></tr></table>	9	7	10	−4	5	8	1	2	−3	6	M1	Attempt at (signed) ranks, ranks may be reversed. May be seen with table before 4(a) .
	9	7	10	−4	5	8	1	2	−3	6			
	[$P = 48$] $Q = 7$	A1	Ignore any errors in positive ranks.										
'7' < 10, reject H ₀ / significant.	M1	Compare <i>their</i> 7 with 10 and appropriate conclusion (may be in terms of H ₁). Note that if <i>their</i> 7 is 10, they should still reject H ₀ . <i>Their</i> 7 must come from attempt at ranks and be the smaller of <i>their</i> P and Q . Test result can be implied by an attempt at an appropriate conclusion in context ignoring hypotheses.											
Sufficient evidence to support the club captains' belief . OR Sufficient evidence to suggest that the anticlockwise route takes longer (to complete on average than the clockwise route).	A1	Correct conclusion from correct working ignoring hypotheses. A0 for $Q = 7$ from incorrect ranks. In context. Level of uncertainty in language used (for example, not 'prove'). Allow '(Some/enough) evidence that/to show/conclude that...'. Do not accept statements such as 'there is insufficient evidence to suggest...'. 6											
4(c)	The population of differences is symmetrical .	B1	Allow underlying/parent distribution for population.										
		1											

Question	Answer	Marks	Guidance
5(a)	[Continuous so] $a \times 4 = b \times (-2)^2 \Rightarrow a = b$	B1	May be awarded for finding b directly (having already found a).
	$\frac{1}{2}(5-1) \times 4a = 0.75$ or $\int_1^5 a(x-1)dx = 0.75 \Rightarrow 8a = 0.75$ OR $\int_5^7 b(x-7)^2 dx = 0.25 \Rightarrow \frac{8}{3}b = 0.25$ OR $\int_1^5 a(x-1)dx + \int_5^7 a(x-7)^2 dx = 1 \Rightarrow 8a + \frac{8}{3}a = 1$	M1	Use of area of triangle = 0.75 to form an equation in a . May see an attempt at integral leading to $\frac{1}{2}a(4^2 - 0^2) = \frac{3}{4}$. Some working may be present on sketch. OR Use of area of upper tail = 0.25 to form an equation in b . OR Use of total area = 1 to form an equation in a (or in b).
	$a = \frac{3}{32}$ OR $b = \frac{3}{32}$	A1	AG, convincingly obtained, no errors seen.
		3	
5(b)	$[F(x) = \begin{cases} 0 & x < 1 \\ \frac{3}{64}(x-1)^2 & 1 \leq x \leq 5 \\ \frac{1}{32}(x-7)^3 + 1 & 5 \leq x \leq 7 \\ 1 & x > 7 \end{cases}]$	M1	Attempt to integrate both parts, all powers correct, condone missing c , ignore domain.
		A1	Both parts fully correct. Ignore domain. For reference: $\frac{3}{64}(x-1)^2 = \frac{3}{64}x^2 - \frac{3}{32}x + \frac{3}{64}$, $\frac{1}{32}(x-7)^3 + 1 = \frac{1}{32}x^3 - \frac{21}{32}x^2 + \frac{147}{32}x - \frac{311}{32}$.
		B1	Correct domains. Must cover all reals, may overlap at boundaries. Must have 0 for $x < 1$ and 1 for $x > 7$.
		3	

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Question	Answer	Marks	Guidance
5(c)	$[G(y) = \begin{cases} 0 & y < 3 \\ -\frac{1}{32}(3-y)^3 & 3 \leq y \leq 5 \\ 1 - \frac{3}{64}(9-y)^2 & 5 \leq y \leq 9 \\ 1 & y > 9 \end{cases}]$	*M1	Expression for $F(10-y)$ seen.
		DM1	Use of $1 - F(10-y)$ in both parts.
		A1	Both parts correct, ignore domain. May be unsimplified. ISW if seen correct in working. For reference: $-\frac{1}{32}(3-y)^3 = \frac{1}{32}(y^3 - 9y^2 + 27y - 27),$ $1 - \frac{3}{64}(9-y)^2 = \frac{1}{64}(-3y^2 + 54y - 179).$
		B1	Correct domain must be in terms of y . Domain may overlap at boundaries. Must have 0 for $y < 3$ and 1 for $y > 9$.
		4	

Question	Answer	Marks	Guidance
6(a)	$H_0: \mu_A = \mu_B, \quad H_1: \mu_A \neq \mu_B$	B1	If in words, must have population means .
	$\Sigma(x - \bar{x})^2 = '9778' - \frac{'270'^2}{8} [= 665.5]$	B1	For correct sum and sum of squares for either x or y . May be implied, for example, by either 95.1 or 56.75 seen.
	$\Sigma(y - \bar{y})^2 = '6079' - \frac{'225'^2}{9} [= 454.0]$	M1	Correct form for both expressions using <i>their</i> values. May be embedded.
	$s_p^2 = \frac{'665.5' + '454.0'}{8 + 9 - 2}$	M1	Correct form of pooled variance using <i>their</i> values.
	$\frac{2239}{30} [= 74.633]$	A1	AWRT 74.6, implied by 2.08.
	$(t =) \frac{\frac{'270'}{8} - \frac{'225'}{9}}{\sqrt{'74.633'(\frac{1}{8} + \frac{1}{9})}}$	M1	Must be in correct form using <i>their</i> s .
	2.084	A1	AWRT 2.08
	'2.084' > 1.753, reject H_0 / significant.	M1	Compare <i>their</i> ± 2.084 with ± 1.753 (consistent signs) and appropriate conclusion (may be in terms of H_1). <i>Their</i> ± 2.084 must come from an attempt at standardising. Test result can be implied by an attempt at an appropriate conclusion in context.

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Question	Answer	Marks	Guidance
6(a)	Sufficient evidence to suggest that there is a difference in the (average) percentage of waste (that is recycled in City <i>A</i> and City <i>B</i>).	A1	Correct conclusion from correct working, ignoring hypotheses. In context. Level of uncertainty in language used (for example, not ‘prove’). Allow ‘(Some/enough) evidence that/to show/conclude that...’. Do not accept statements such as ‘there is insufficient evidence to suggest...’.
		9	Combined <i>s</i> max 5/9 B1B1M1M0A0M1A0M1A0
6(b)	Population of percentage of household waste recycled in each city are normal .	B1	B1 for underlying/parent distributions are normal. B0 for population mean normally distributed. B0 for (population of/underlying) data/sample is normal. Allow singular, for example, population is normal. Allow use of <i>A</i> and <i>B</i> to denote population / random variable. Must not refer to differences.
		1	